Exponential Growth and Decay (24E)

Exponential Growth and Decay Models (Financial)

Investigate

Real life situation 1) The National Association Realtors estimates that, on average, the price of a house doubles every ten years. Tony's grandparents bought a house in 1960 for $10,000. Assume that the trend identified by the National Association of Realtors applies to Toney's Grandparents' house.

Year	Decades Since 1960	Value of House	Difference Between Values of Consecutive Decades	Ratio of Values of Consecutive Decades
1960				
1970				
1980				
1990				
2000				
2010				
2020				

*Graph on a graph paper.

a) The value of Tony's grandparents' house is growing exponentially because it is multiplied by a constant factor for each decade. What is the constant factor?

Situation 2) You received $10,000 in gift money when you graduated from college. You deposit the money into an account that pays 5% compound interest annually.

a) Fill out the chart

Year	Account Balance at the end of the year
0	$ 10,000
1	$ 10,500
2	$ 11,025
3	
4	
5	

b) Instead of calculating the amount of money in the account after each year, write an expression for each amount of money using $10,000 and repeated multiplication of the constant factor. Then rewrite each expression using exponents.

Year	Account Balance at the end of the year
0	$ 10,000
1	($ 10,000) (1.05) = ($10,000) $(1.05)^1$
2	($ 10,000) (1.05) (1.05) = ($10,000) $(1.05)^2$
3	($ 10,000) (1.05) (1.05) (1.05) = ($10,000) $(1.05)^3$
4	($ 10,000) (1.05) (1.05) (1.05) (1.05) = ($10,000) $(1.05)^4$
5	
6	
7	
8	
9	
10	

c) Write an expression to represent the amount of money in the account at the end of Year 8.

d) How much money would you have in the account after 20 years? After 30 years?

e) **Express regularity in repeated reasoning.** Write an expression to represent the amount of money in the account after t years.

f) Write your expression as a function *m(t)*, where *m(t)* is the total amount of money in your account after *t* years.

g) Use the data from the table in Item2 to graph the function.

h) Identify the reasonable domain and range.

i) Your future plans include purchasing a home. You estimates that you will need at least $20,000 for a down payment. Determine the year is which you will have enough funds in your account for the down payment.

At the same time that you open your account, Your friend Rissah deposits $10,000 in an account with an annual compound interest rate of 6%.

j) Write a new function to represent the total funds in Rissah's account, *f(t)*, after *t* years.

k) Predict how the graph of Rissah's account balance will differ from the graph of your account balance.

l) Over a long period of time, does the initial deposit or the interest rate have a greater amount of money in an account that has interest compounded yearly? Explain your reasoning.

Situation 3)

Most savings institutions offer compounding intervals other than annual compounding. For example, a bank that offers quarterly compounding computes interest on an account every quarter; that is, at the end of every 3 months. Instead of computing the interest once each year, interest is computed four times each year. If a bank advertises that it is offering 8% interest *compounded quarterly*, 8% is not the actual growth factor. Instead, the bank will use $\frac{8\%}{4} = 2\%$ to determine the quarterly growth factor.

a) What is the quarterly interest rate for an account with an annual interest rate of 5% *compounded quarterly*?

Suppose that you invested your $10,000 in the account described in item #3a).

b) In the table below, determined your account balance after the specified times since her initial deposit.

Time Since Initial Deposit	Number of Times Interest Has Been Compounded	Account balance
3 months		
6 months		
9 months		
1 year		
4 years		
t years		

c) Write a function A(t) to represent the balance in your account after t years.

d) Calculate the balance in your account after 20 years.

e) For the compounding periods given below, write a function to represent the balance in your account after t years. Then calculate the balance in the account after 20 years. You are investing $10,000 at a rate of 5% annual compound interest.

a. Yearly:	
b. Quarterly:	
c. Monthly:	
d. Daily (assume there are 365 days in a year)	

General Formula for compounded interest

e. Yearly:	
f. Quarterly:	
g. Monthly:	
h. Daily (assume there are 365 days in a year)	

APPLY/Fluency

1) You deposit $2000 to an account that pays 8% annual interest for 4 years. How much does the account earn Monthly? Semi-Annually? Continuously?

Monthly   **Semi-Annually**   **Continuously**

2) You deposit $500 to an account that pays 5%. How long will it take for your money to triple if its compounded monthly, annually, and continuously?

Monthly   **Annually**   **Continuously**

Solve each exponential model. Round all answers to two decimal places.

3) Find the accumulated value of an investment of $10,000 for 4 years at an interest rate of 3.5% if the money is **a)** compounded semiannually **b)** compounded monthly **c)** compounded daily and **d)** compounded continuously.

4) Suppose that you have $6000 to invest. Which investment yields the greater return over 4 years: 8.25% compounded quarterly or 8.3% compounded semiannually?

5) Find the amount of time it would take to get $5000 on a $3000 investment if it is invested at 6% compounded quarterly.

6) Find the accumulated amount of an investment of $9000 at 4% compounded monthly for 6 years.